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Exact half-BPS flux solutions in M-theory II: global solutions asymptotic to $AdS_7 \times S^4$*

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Abstract: General local half-BPS solutions in M-theory, which have $\mathrm{SO}(2,2) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry and are asymptotic to $AdS_7 \times S^4$, were constructed in exact form by the authors in [arXiv:0806.0605]. In the present paper, suitable regularity conditions are imposed on these local solutions, and corresponding globally well-defined solutions are explicitly constructed. The physical properties of these solutions are analyzed, and interpreted in terms of the gravity duals to extended 1+1-dimensional half-BPS defects in the 6-dimensional CFT with maximal supersymmetry.

Keywords: AdS-CFT Correspondence, M-Theory.

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Contents

1. Introduction

One realization of the AdS/CFT correspondence [1 − 3] in M-theory is the duality of the $AdS_7 \times S^4$ vacuum and the 6-dimensional CFT which is obtained by a decoupling limit of the M5 brane world-volume theory [4 − 6]. The nonabelian world-volume theory of multiple M5-branes is presently unknown and the 6-dimensional CFT has been formulated in the light cone gauge [7]. One interesting class of deformations in this theory is given by the insertion of local half-BPS chiral operators, where half-BPS means the operators preserve sixteen of the thirty-two supersymmetries. The gravitational duals of these operators are the half-BPS solutions of Lin, Lunin, and Maldacena [8].

In our recent paper [9] (see also [10 − 13] for earlier work), new exact solutions of 11-dimensional supergravity were constructed which preserve sixteen of the thirty-two supersymmetries. In addition, the solutions preserve a $\mathrm{SO}(2, 2) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$ bosonic symmetry. Correspondingly, the 11-dimensional metric is constructed as a warped product of $AdS_3 \times S^3 \times S^3$ over a 2-dimensional base space Σ. These solutions can be interpreted as the gravity duals of extended supersymmetric defects in the CFT. The solutions are local in the sense that for a bosonic background, the vanishing of gravitino variation as well as the bosonic equations of motion and Bianchi identities are satisfied point wise, except at possible singularities.

In general, the local solutions of [9] contain a large variety of solutions many of which contain singularities. An important problem is to pick out solutions which are asymptotic to either $AdS_4 \times S^7$ or $AdS_7 \times S^4$ and everywhere regular so that the supergravity approximation is valid. In particular this requires one to examine the global structure of the solutions. Similar analysis have been carried out in Type IIB supergravity in [8, 15, 18]. An amazing result in all cases is that the regularity conditions in addition to the general local solution admit a superposition principle for the half-BPS objects in the theory. We expect such a principle to emerge from the analysis here. There are two distinct classes of solutions which were found in [9]:

Case I contains solutions asymptotic to $AdS_4 \times S^7$. The superalgebra of symmetries is $\mathrm{OSp}(4^*|2) \times \mathrm{OSp}(4^*|2)$. The super Lie algebra $\mathrm{OSp}(4^*|2)$ is a particular real form of $\mathrm{OSp}(4|2)$. In the classification of [17] this solution is case IV of table 11. M-theory on $AdS_4 \times S^7$ is dual to a 3-dimensional CFT which is obtained by a decoupling limit of the world-volume theory of M2 branes. The local BPS solution is dual to $1+1$-dimensional conformal defects in the 3-dimensional CFT, analogous to the half-BPS defect solutions obtained in type IIB string theory [14, 15] (see also [16] for earlier work).

Case II contains solutions asymptotic to $AdS_7 \times S^4$. The bosonic isometries together with the supersymmetries form a superalgebra which is given by $\mathrm{OSp}(4|2,\mathbf{R}) \times \mathrm{OSp}(4|2,\mathbf{R})$. The supergroup $\mathrm{OSp}(4|2,\mathbf{R})$ is a different real form of $\mathrm{OSp}(4|2)$ than the one appearing in case I. In the classification of [17] this solution is case VII of table 12. M-theory on $AdS_7 \times S^4$ is dual to a 6-dimensional CFT which is obtained by a decoupling limit of the world-volume theory of M5 branes. The local BPS solution is dual to $1+1$-dimensional conformal defects in the 6-dimensional CFT, analogous to the half-BPS Wilson loop solutions obtained in type IIB string theory [18] (see also [12, 19] for earlier work).

The local solutions presented in [9] are very similar for the case I and II as the underlying integrable system is the same. However, the analysis of the regularity and the global structure is quite different. In this paper we will focus on the solution of case II. The analysis of the regularity and global structure for case I will be analyzed in a separate paper [20].

The structure of the paper is as follows. In section 2 the features of the local solution for case II which are important for the present paper will be reviewed. In section 3 the boundary conditions on the solution implied by regularity are analyzed. A general solution which satisfies suitable boundary condition is constructed and it is shown that the solution is regular everywhere. In section 4 the global structure of the solutions as well as its interpretation in terms of the dual 6-dimensional conformal field theory is discussed. In appendix A detailed proof of the regularity of our solution is presented.

2. Summary of local solution

In this section we review the local half-BPS solution of [9]. Derivations and more calculational details can be found in that paper. The 11-dimensional metric is a fibration of $AdS_3 \times S^3 \times S^3$ over a 2-dimensional Riemann surface Σ,

$$ds^2 = f_1^2 ds^2_{AdS_3} + f_2^2 ds^2_{S_2^3} + f_3^2 ds^2_{S_3^3} + ds^2_\Sigma \tag{2.1}$$

The four form field strength is given by

$$F_4 = g_{1a}\ \omega_{AdS_3} \wedge e^a + g_{2a}\ \omega_{S^3_2} \wedge e^a + g_{3a}\ \omega_{S^3_3} \wedge e^a \tag{2.2}$$

Here ω_{AdS_3} and $\omega_{S^3_{2,3}}$ are the volume forms on AdS_3 and $S^3_{2,3}$ respectively. In addition $e^a, a = 1, 2$ is the vielbein on Σ. It is always possible to choose local complex coordinates $w, \bar{w}$ on the Riemann surface Σ such that the 2-dimensional metric in (2.1) is given by

$$ds^2_\Sigma = 4\rho^2\ |dw|^2 \tag{2.3}$$

The metric factors f_1, f_2, f_3, ρ, as well as the flux fields $g_{1a}.g_{2a}$, and g_{3a} only depend on Σ.

The Ansatz respects $\mathrm{SO}(2,2) \times \mathrm{SO}(4) \times \mathrm{SO}(4)$ symmetry which can be interpreted as the symmetries of a 1+1-dimensional conformal defect in the 6-dimensional M5 brane CFT. The condition that 16 supersymmetries are unbroken is equivalent, for a purely bosonic background, to the statement that the gravitino supersymmetry variation $\delta_\epsilon \Psi_M$ vanishes for 16 linearly independent supersymmetry variation parameters.

In [9] the BPS conditions were solved, and it was shown that the half-BPS solution is completely determined by the choice of a 2-dimensional Riemann surface Σ, a real harmonic functions $h(w, \bar{w})$ on Σ and a complex function $G(w, \bar{w})$ which is a solution of the following linear equation,

$$\partial_w G = \frac{1}{2}(G + \bar{G})\partial_w \ln h \tag{2.4}$$

In order to express the local half-BPS solution in terms of G and h it is useful to define the following quantity

$$W^2 = -|G|^4 - (G - \bar{G})^2 \tag{2.5}$$

The metric factors in (2.1) are then given by

$$\begin{aligned}
f_1^6 &= 4h^2 \frac{(1 - |G|^2)}{W^4}\Big(|G - \bar{G}| + 2|G|^2\Big)^3 \\
f_2^6 &= 4h^2 \frac{(1 - |G|^2)}{W^4}\Big(|G - \bar{G}| - 2|G|^2\Big)^3 \\
f_3^6 &= \frac{h^2 W^2}{16(1 - |G|^2)^2}
\end{aligned} \tag{2.6}$$

The metric factor in (2.3) is given by

$$\rho^6 = \frac{(\partial_w h \partial_{\bar{w}} h)^3}{16h^4}(1 - |G|^2)W^2 \tag{2.7}$$

The fluxes g_i are defined by conserved currents as follows

$$\begin{aligned}
(f_1)^3 g_{1w} = -\partial_w b_1 &= 2(j^+_w + j^-_w) \\
(f_2)^3 g_{2w} = -\partial_w b_2 &= -2(j^+_w - j^-_w) \\
(f_3)^3 g_{3w} = -\partial_w b_3 &= \frac{1}{8}\, j^3_w
\end{aligned} \tag{2.8}$$

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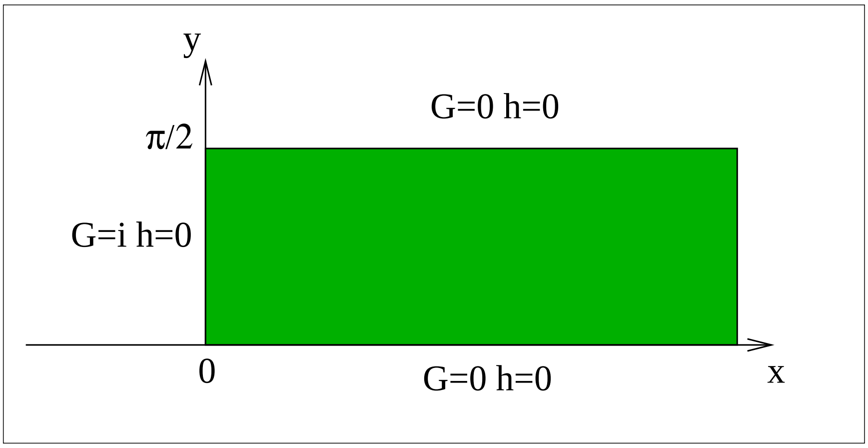

Figure 1: Σ and boundary conditions for $AdS_7 \times S^4$ solution

where the conserved currents can be expressed in a compact way by defining

$$J_w = \frac{h}{G+\bar{G}}\Big(\bar{G}(G - 3\bar{G} + 4G\bar{G}^2)\partial_w G + G(G+\bar{G})\partial_w \bar{G}\Big) \tag{2.9}$$

and are given by

$$\begin{aligned} j_w^+ &= 2i\, J_w\Big((G-\bar{G})^2 - 4G^3\bar{G}\Big)W^{-4} \\ j_w^- &= 2\,GJ_w\Big(-2G\bar{G} + 3\bar{G}^2 - G^2 + 4G^2\bar{G}^2\Big)W^{-4} \\ j_w^3 &= 3\partial_w h\,\frac{W^2}{G(1-G\bar{G})} - 2J_w\,\frac{(1+G^2)}{G(1-G\bar{G})^2} \end{aligned} \tag{2.10}$$

It was shown in [9] that the equations of motion of as well as the Bianchi identities are satisfied for a harmonic h and a G which solves (2.4).

The simplest solution is the maximally symmetric $AdS_7 \times S^4$ itself. The Riemann surface is the half strip $\Sigma = \{(x,y), x \geq 0, 0 \leq y \leq \pi/2\}$. Denoting the holomorphic coordinate as $w = x + iy$, the functions h and G are given by

$$\begin{aligned} h &= -i(\cosh(2w) - \cosh(2\bar{w})) \\ G &= -i\frac{\sinh(w-\bar{w})}{\sinh(2\bar{w})} \end{aligned} \tag{2.11}$$

Plugging this into (2.6) the metric factors become

$$f_1 = 2\mathrm{ch}(x) \qquad f_2 = 2\mathrm{sh}(x) \qquad f_3 = \sin(2y) \qquad \rho = 1 \tag{2.12}$$

Note that the Riemann surface Σ has three boundary components. The boundary is characterized by the vanishing of the harmonic function $h = 0$. Furthermore, taking $y = 0$ or $y = \pi/2$, we find $G = 0$, while taking $x = 0$, we find $G = +i$. So G takes the values 0 and $+i$ on the boundary of Σ. The boundary of $AdS_7 \times S^4$ on the other hand is located at $x = \infty$.

3. Regularity

In this section, we analyze the regularity conditions and the global structure of the solution. We look for solutions which are everywhere regular and which are asymptotic to $AdS_7 \times S^4$. This leads to the following three assumptions for the geometry.

1. The boundary of the 11-dimensional geometry is asymptotic to $AdS_7 \times S^4$.
2. The metric factors are finite everywhere, except at points where the geometry becomes asymptotically $AdS_7 \times S^4$, in which case the AdS_3 metric factor and one of the sphere metric factors diverge.
3. The metric factors are everywhere non-vanishing, except on the boundary of Σ, in which case at least one sphere metric factors vanishes. In addition, both sphere metric factors may vanish only at isolated points.

The second requirement guarantees that all singularities in the geometry are of the same type as $AdS_7 \times S^4$. The third requirement guarantees that the boundary of Σ corresponds to an interior line in the 11-dimensional geometry.

It follows from (2.6) that a particular combination of metric factors is very simple

$$(f_1 f_2 f_3)^6 = h^6 \tag{3.1}$$

The metric factor f_1 is positive definite and cannot vanish [9]. Hence the condition $h = 0$ (which defines a 1-dimensional subspace in Σ) occurs if and only if at least one of the metric factors for the spheres f_2 or f_3 vanishes. It follows from assumption 3, that $h = 0$ defines the boundary of Σ.

Note that the equation for G (2.4) is covariant under conformal reparamaterizations. This freedom allows one to choose local conformal coordinates[1]

$$u = r + i\ s, \qquad r = h(w, \bar{w}), \qquad s = \tilde{h}(w, \bar{w}) \tag{3.2}$$

Here, $\tilde{h}$ is the harmonic function dual to h so that u is holomorphic, i.e. $\partial_{\bar{u}}(h + i\tilde{h}) = 0$. The domain of the new conformal coordinate u is the right half plane and the boundary of Σ is at $r = 0$, i.e. the vertical axis.

In the coordinates r, s, it is useful to decompose G into its real and imaginary parts,

$$G(r, s) = G_r(r, s) + iG_s(r, s) \tag{3.3}$$

for G_r, G_s real functions. The real and imaginary parts of equation (2.4) are respectively,

$$\partial_r G_r + \partial_s G_s = \frac{G_r}{r} \tag{3.4}$$

$$\partial_r G_s - \partial_s G_r = 0 \tag{3.5}$$

Equation (3.5) can be solved in terms of a single real potential

$$G_r = \partial_r(r\Psi) \qquad\qquad G_s = \partial_s(r\Psi) \tag{3.6}$$

[1] We use a slightly different notation: the coordinate s in this paper was called x in [9].

Equation (3.4) becomes a second order partial differential equation on Ψ,

$$\left(\partial_s^2 + \partial_r^2 + \frac{1}{r}\partial_r - \frac{1}{r^2}\right)\Psi(r,s) = 0 \tag{3.7}$$

The general local solution of (3.7) can be obtained by a Fourier transformation with respect to s, which produces an ordinary differential equation which can be solved by [9]

$$\Psi(r,s) = \int_{-\infty}^{\infty} \frac{dk}{2\pi}\, \psi_2(k) K_1(kr) e^{-iks} \tag{3.8}$$

Here K_1 is the modified Bessel function of the second kind. There is a second linearly independent solution of the form (3.8) which involves the modified Bessel function of the first kind $I_1(kr)$. However this solution has the wrong behavior for large r and fails to obey the regularity condition $|G|^2 \leq 1$. In [9] an explicit expression for Ψ and G was found by defining $C_2(v)$

$$C_2(v) = \int_0^{\infty} \frac{dk}{2\pi}\, \psi_2(k)\, e^{-kv} \tag{3.9}$$

and using the following integral representation of K_1

$$K_1(kr) = \int_1^{\infty} \frac{t\,dt}{\sqrt{t^2-1}}\, e^{-tkr} \tag{3.10}$$

Using (3.8)- (3.10) Ψ can be expressed in terms of C_2

$$\Psi(r,s) = \int_1^{\infty} \frac{t\,dt}{\sqrt{t^2-1}} \Big(C_2(tr+is) + C_2(tr+is)^* \Big) \tag{3.11}$$

and (3.6) can be used to write G as follows

$$G(r,s) = r\int_1^{\infty} \frac{dt}{\sqrt{t^2-1}} \Big((1-t)C_2'(tr+is) + (1+t)C_2'(tr+is)^* \Big) \tag{3.12}$$

3.1 Boundary conditions on G

In this section we analyze the boundary conditions G has to satisfy at $h=0$ or in the r,s coordinates at $r=0$ in order for the solution to be regular near the boundary. It will be useful to have the following expressions for the metric factors obtainable from (2.6)

$$\begin{aligned} (f_1 f_2)^3 &= -4r^2 \frac{(1-|G|^2)}{W} \\ f_3^3 &= -\frac{rW}{4(1-|G|^2)} \end{aligned} \tag{3.13}$$

The behavior near the boundary (the regularity in the bulk of Σ will be discussed in the next section) is exhibited by expanding G_r, G_s, at fixed s, in a power series in r,

$$\begin{aligned} G_r &= \gamma_1\, r + \gamma_3\, r^3 + \gamma_5\, r^5 + \mathcal{O}(r^7) \\ G_s &= \gamma_0 + \gamma_2\, r^2 + \gamma_4\, r^4 + \mathcal{O}(r^6) \end{aligned} \tag{3.14}$$

where the γ_i are all functions of s. Note that the reality of the solution implies that G is bounded $|G| \leq 1$ and hence no negative powers of r can appear in the series expansion (3.14). Equation (3.4) and (3.5) impose the vanishing of even/odd powers of r in G_r/G_s respectively. Furthermore these equations impose differential equations in s between the different $\gamma_i(s)$ but these relations will not be needed in the following. The following power series expansions will be useful in the following analysis,

$$\begin{aligned} W^2 &= 4\gamma_0^2(1-\gamma_0^2) + 8(\gamma_0\gamma_2 - 2\gamma_0^3\gamma_2 - \gamma_0^2\gamma_1^2)r^2 + \mathcal{O}(r^4) \\ 1-|G|^2 &= (1-\gamma_0^2) - (\gamma_1^2 + 2\gamma_0\gamma_2)r^2 + \mathcal{O}(r^4) \end{aligned} \tag{3.15}$$

When $\gamma_0 \neq \pm 1$, we have $|G| \neq 1$ as $r \to 0$. From (3.13), the product $f_1 f_2$ goes to zero as $r \to 0$ and the geometry will be singular unless $W \sim r^2$. It follows from (3.15) that one has to choose $\gamma_0 = 0$ in order to avoid a singularity. If this is the case, f_2 will remain finite, while $f_3 \to 0$ so that the volume of the sphere S_3^3 tends to zero. Comparing with the $AdS_7 \times S^4$ solution, this behavior is associated with the $y = 0, \pi/2$ boundary component of (2.11).

When $\gamma_0 = \pm 1$, we will have $f_2 \to 0$ while f_3 will remain finite, as long as the conditions $\gamma_1 \neq \mp 2\gamma_2$ and $\gamma_1^2 \neq \mp\gamma_2$ are satisfied. If this is the case, the volume of the sphere S_2^3 will tend to zero. Comparing with the $AdS_7 \times S^4$ solution, this behavior is associated with the $x = 0$ boundary component of (2.11).

In summary we have the following boundary conditions on G and the metric factors

$$\begin{array}{lllll} G(0,s) = 0 & \Leftrightarrow & f_2 \neq 0 & f_3 = 0, & \mathrm{Vol}(S_3^3) \to 0 \\ G(0,s) = \pm i & \Leftrightarrow & f_2 = 0 & f_3 \neq 0 & \mathrm{Vol}(S_2^3) \to 0 \end{array} \tag{3.16}$$

3.2 General regular solution

We first parameterize the boundary conditions for G at $r = 0$ which leads to solutions satisfying regularity conditions 1 to 3, listed at the beginning of section 3. As we shall see, a further condition needs to be imposed to guarantee the absence of singularities in the bulk, namely the values of G in the second line of (3.16) will be restricted to be either all positive or all negative.

General boundary conditions for G are given by the choice of $g+2$ points on the s axis,

$$-\infty < a_1 < b_1 < a_2 < b_2 < \cdots < a_{g+1} < b_{g+1} < \infty \tag{3.17}$$

The $AdS_7 \times S^4$ solution corresponds to $g = 0$. There are two kinds of intervals which are distinguished by the boundary condition for $G(0,s)$.

$$\begin{array}{lll} s \in [-\infty, a_1] & G(s,0) = 0 & \\ s \in [a_n\,,\, b_n] & G(s,0) = i\eta_n, & n = 1,2,\cdots g \\ s \in [b_n, a_{n+1}] & G(s,0) = 0, & n = 1,2,\cdots g \\ s \in [b_{g+1}, \infty] & G(s,0) = 0 & \end{array} \tag{3.18}$$

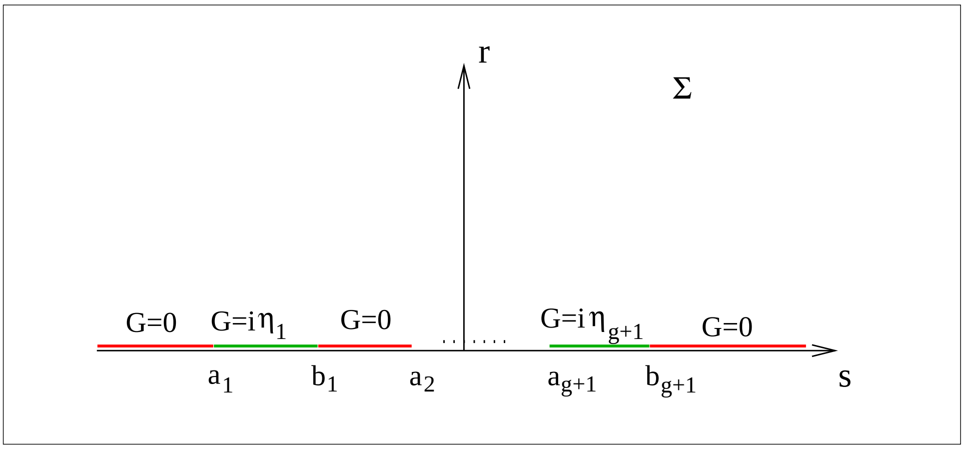

Figure 2: The surface Σ and boundary conditions for a general regular solution.

where $\eta_n = \pm 1$. The boundary conditions can be implemented as follows:

$$G(s,0) = \sum_{n=1}^{g+1} i\eta_n \Big(\Theta(s-a_n) - \Theta(s-b_n) \Big) \tag{3.19}$$

where $\Theta(s)$ is the step function. Since on each segment, $G(s,0)$ can take only the values $0, \pm i$, no two bumps can "overlap", and this forces the a_n and b_n to alternate as in (3.17).

It remains to calculate $\psi_2(k)$ from the boundary condition in (3.19). To this end, we first take the Fourier transform in s with $\ell > 0$, of (3.6) with (3.8) plugged in

$$\int_{-\infty}^{\infty} ds \; e^{+i\ell s} \; \partial_s \Big(r\Psi(s,r) \Big) = -i\ell r \psi_2(\ell) K_1(\ell r) \tag{3.20}$$

whose $r \to 0$ limit is simply obtained using the asymptotics of K_1, and we find,

$$\int_{-\infty}^{\infty} ds \; e^{+i\ell s} \; \lim_{r\to 0} \partial_s \Big(r\Psi(s,r) \Big) = -i\psi_2(\ell) \tag{3.21}$$

Using the boundary condition of (3.19), we thus have

$$\psi_2(\ell) = \sum_n i\eta_n \int_{b_n}^{a_n} ds \; e^{i\ell s} = \sum_n \eta_n \frac{e^{i\ell a_n} - e^{i\ell b_n}}{\ell} \tag{3.22}$$

The Fourier transform can be done exactly

$$C_2(v) = -\frac{1}{2\pi} \sum_n \eta_n \ln \left(\frac{v - ia_n}{v - ib_n} \right) \tag{3.23}$$

The expression for $G(s,r)$ may then be obtained using the integral representation (3.12)

$$G(s,r) = \int_1^{\infty} \frac{dt}{\sqrt{t^2-1}} \Big(r(1-t) C_2'(tr+is) + r(1+t) C_2'(tr+is)^* \Big) \tag{3.24}$$

After some simplifications, we obtain,

$$G(s,r) = -\frac{1}{2\pi}\sum_{n=1}^{g+1}\eta_n\int_1^\infty \frac{dt}{\sqrt{t^2-1}}\left[\frac{1+i\alpha_n}{t+i\alpha_n}+\frac{1+i\alpha_n}{t-i\alpha_n}-\frac{1+i\beta_n}{t+i\beta_n}-\frac{1+i\beta_n}{t-i\beta_n}\right] \quad (3.25)$$

where the quantities α_n and β_n are defined as

$$\alpha_n = (s-a_n)/r, \qquad \beta_n = (s-b_n)/r, \quad n = 1,2,\cdots g+1 \quad (3.26)$$

are both real. Next we use the integral formula

$$\int_1^\infty \frac{dt}{\sqrt{t^2-1}}\left(\frac{1}{t+z}+\frac{1}{t-z}\right) = \frac{\pi}{\sqrt{1-z^2}} \quad (3.27)$$

The result is an algebraic expression for G which solves (3.4) and satisfies the boundary condition (3.19) and is given by

$$G(s,r) = -\frac{1}{2}\sum_{n=1}^{g+1}\eta_n\left(\frac{r+is-ia_n}{\sqrt{r^2+(s-a_n)^2}}-\frac{r+is-ib_n}{\sqrt{r^2+(s-b_n)^2}}\right) \quad (3.28)$$

It is easy to see that (3.28) is equal to (3.19) in the limit $r \to 0$.

3.3 Regularity in the bulk

In the remainder of this section we give an argument that the general solution (3.28) is regular everywhere. Note that the general solution was constructed in section 3.2 by demanding that the geometry is regular at the boundary of Σ.

The general solution (3.28) should also approach the $AdS_7 \times S^4$ boundary asymptotically as $r \to \infty$. The expression for $AdS_7 \times S^4$. given in (2.11) can be recovered from the general regular solution (3.28) by setting $g = 0$, $\eta_1 = -1$, $a_1 = -2$ and $b_1 = 2$.

$$\begin{aligned} G(s,r) &= \frac{r+is+2i}{2|r+is+2i|}-\frac{r+is-2i}{2|r+is-2i|} \\ &= -i\frac{\mathrm{sh}(w-\bar{w})}{\mathrm{sh}(2\bar{w})} \end{aligned} \quad (3.29)$$

where we have used the coordinates $r = 2\sin(2y)\sinh(2x)$ and $s = 2\cos(2y)\sinh(2x)$. The boundary of AdS_7 is reached by taking $x \to \infty$. Using the same coordinate change for the general solution it is easy to see that (3.28) approaches $AdS_7 \times S^4$ in the limit $x \to \infty$.

It remains to show that the geometry is regular in the interior for Σ. Since $h = r$ in the chosen coordinate system the regularity of the solution requires that away from the boundary W is required to satisfy the strict inequality $W^2 > 0$. Note that this condition automatically guarantees that we also have $1 - |G|^2 > 0$. Furthermore the relation

$$W^2 = -4|G|^4 - (G-\bar{G})^2 = (|G-\bar{G}| - 2|G|^2)(|G-\bar{G}| + 2|G|^2) \quad (3.30)$$

shows that if $W^2 > 0$ then individually $(|G-\bar{G}| - 2|G|^2) \neq 0$ and $(|G-\bar{G}| + 2|G|^2) \neq 0$. Examining the explicit formula for the metric factors (2.6) one can see that they are then

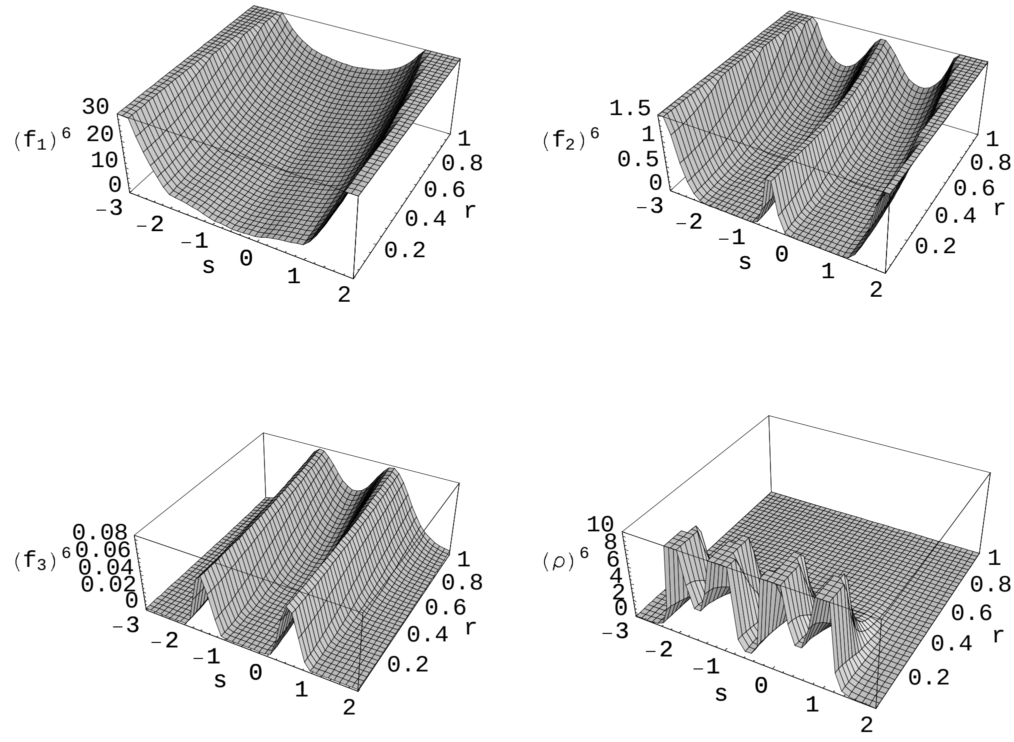

Figure 3: Metric factors for a $g = 1$ solution.

finite and the geometry is thus regular. It is shown in appendix A if the a_n, b_n obey the ordering (3.17) and the η_n are all either $+1$ or -1, that $W^2 > 0$ in the upper half plane and hence the solution is regular everywhere.

To see the converse, that is if $W^2 = 0$ in the bulk then the solutions is singular, we first note that for W^2 to vanish, either $(|G - \bar{G}| - 2|G|^2)$ or $(|G - \bar{G}| + 2|G|^2)$ must vanish. Taking the ratio of f_1 and f_2 in (2.6) we see that one of them must either be vanishing or be infinite resulting in a singular geometry.

3.4 The $g = 1$ solution

In this section, we examine the $g = 1$ solution in detail, specifically we choose the parameters for the general solution (3.28) as follows:

$$g = 1, \qquad a_1 = -2, \quad b_1 = -1, \quad a_2 = 0, \quad b_2 = 1 \tag{3.31}$$

In figure 3, we show the behavior of the metric factors. The sphere metric factors alternatingly vanish as $r \to 0$. While as $r \to \infty$, the metric factors flatten out to those of $AdS_7 \times S^4$.

For ρ there are singularities as we approach $r \to \infty$, but these are coordinate singularities and are due to the conformal transformation we made in order to map the half-strip to the upper half plane.

An important feature of the solutions with $g > 0$ is that there are additional nontrivial four cycles in the geometry. We can illustrate this feature for the $g = 1$ solution (3.31). In

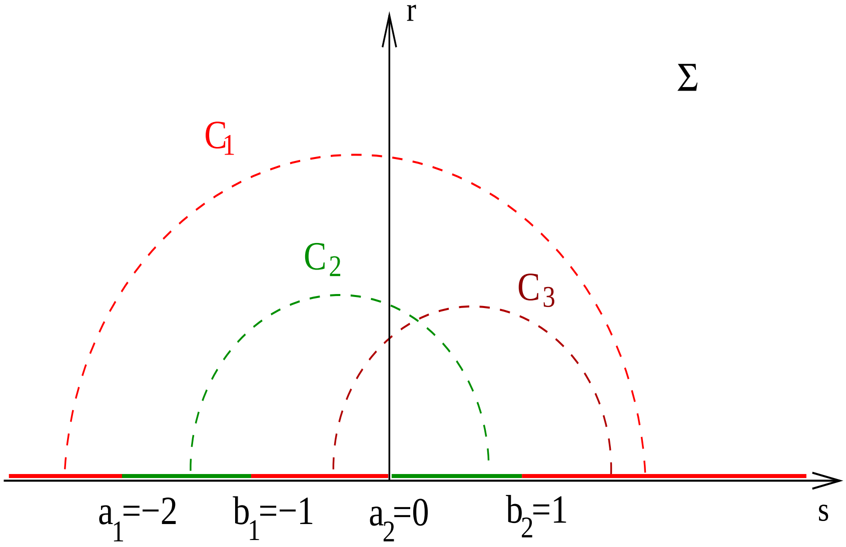

Figure 4: Nontrivial four cycles for the $g = 1$ solution

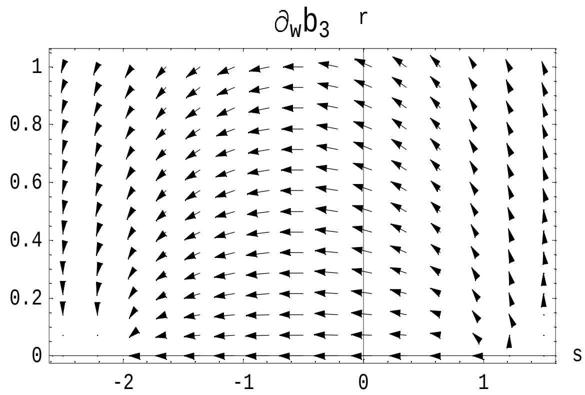

Figure 5: The fluxes for a $g = 0$ solution.

addition to the four cycle C_1 which is already present in the $g = 0$ solution, there are two additional nontrivial four cycles C_2 and C_3.

The behavior of the fluxes for the $g = 1$ solution is very interesting. For comparison purposes we first plot the fluxes (2.8) for the $AdS_7 \times S^4$ solution given by (3.28) with

$$g = 0, \quad a_1 = -2, \quad b_1 = 1 \tag{3.32}$$

Note that the fluxes g_1 and g_2 vanish identically and the only nontrivial flux is g_3. There is only one nontrivial topological cycle forming a four sphere. The integrated flux g_3 is nothing but the non-vanishing four form flux through the four sphere in $AdS_7 \times S^4$.

In figure 6, we plot the fluxes for the $g = 1$ solution (3.31). Due to the complicated form of the currents (2.8) we have not been able to integrate them to analytically obtain a closed form of the fluxes. However it is clear from figure 6 that the $g = 1$ solution has indeed nontrivial flux through the cycles C_2 and C_3 for g_1 and g_2 respectively.

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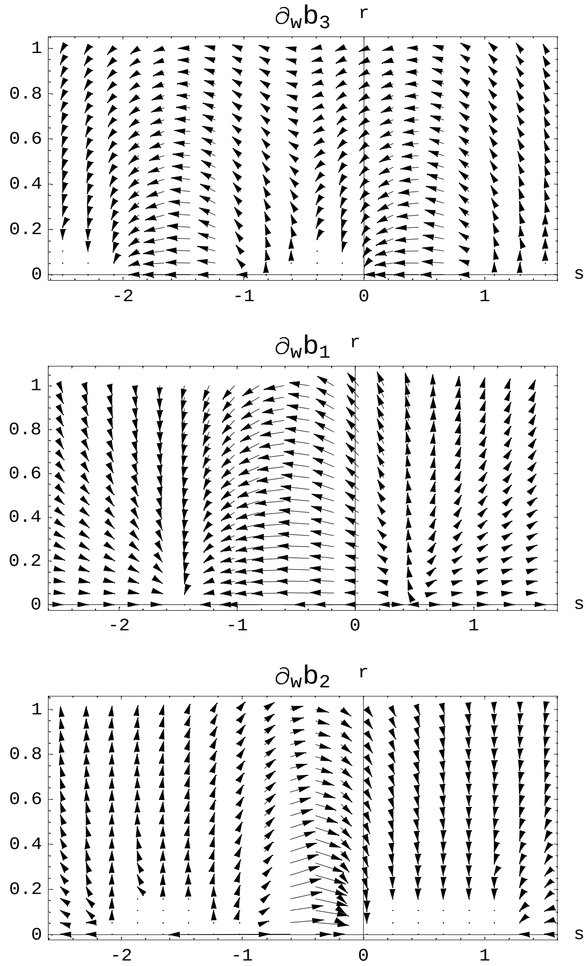

Figure 6: The fluxes for a $g = 1$ solution.

4. Discussion

In the previous section we found a family of regular half-BPS solutions labelled by an integer g and $2g+1$ real moduli. In this section we discuss the interpretation of these solutions from the point of view of the AdS/CFT correspondence. The $AdS_7 \times S^4$ spacetime is obtained as the near horizon limit of a large number of M5 branes. The AdS/CFT duality relates M-theory on this background to the decoupling limit of the M5-brane world-volume theory which defines a 6-dimensional CFT with $(2,0)$ supersymmetry [5, 4].

A first step towards interpreting the solution is to understand the boundary structure. The only region on Σ where the spacetime becomes asymptotically $AdS_7 \times S^4$ is $r \to \infty$. There is however another boundary component since the AdS_3 factor also has a boundary. This can be seen by rewriting the metric (2.1).

$$ds^2 = \frac{1}{z^2}\Big(f_1^2(dz^2 + dx^2 - dt^2) + z^2 f_2^2 ds^2_{S_2^3} + z^2 f_3^2 ds^2_{S_3^3} + z^2 ds^2_\Sigma\Big) \tag{4.1}$$

The boundary of AdS_3 is reached as $z \to 0$ and the boundary metric is obtained by stripping off the (divergent) conformal factor $1/z^2$. The z^2 factor in front of the metric factors of the spheres and Σ implies that the boundary in the limit $z \to 0$ is $1+1$-dimensional space extending in the t, x plane. The $\mathrm{SO}(2,2)$ isometry of the AdS_3 factor corresponds to the conformal symmetry of the $1+1$-dimensional defect, which is contained in the $\mathrm{OSp}(4|2, \mathbf{R}) \times \mathrm{OSp}(4|2, \mathbf{R})$ supergroup of preserved superconformal symmetries. Note that for all values of g and the moduli there is only one defect.

The interpretation of the 1+1-dimensional half-BPS defect from the perspective of the dual CFT is the supersymmetric self dual string solution of the 6-dimensional $(2,0)$ supersymmetric M5-brane world-volume theory [21 − 23] which was constructed in [24]. The selfdual string in the $(2,0)$ theory can also be interpreted as the boundary of an open M2 brane which ends on the M5-brane [25, 26] Unfortunately the action for multiple membranes is not well understood and the selfdual string soliton solution has only been derived for the abelian case of a single 5-brane.

There is a strong analogy of the selfdual string defect with the BPS-Wilson loop in Type IIB string theory. While the details of the supergravity solution are somewhat different the general structure of the half-BPS flux solution and its moduli space presented in section 3.2 is intriguingly similar to the Type IIB supergravity flux solutions dual to BPS Wilson loops which was found in [18].

The BPS Wilson loop in $AdS_5 \times S^5$ also has a probe description. The original proposal [35, 36] identified the Wilson loop in the fundamental representation with a fundamental string with AdS_2 world-volume inside AdS_5. BPS-Wilson loops in higher rank symmetric representation and are identified with a probe D3 brane with electric flux with $AdS_2 \times S^2$ world-volume inside AdS_5. BPS-Wilson loops in higher rank anti-symmetric representation and are identified with a probe D5 brane with electric flux with $AdS_2 \times S^4$ world-volume inside $AdS_5 \times S^5$ [27, 28].

The 1+1-dimensional BPS defect in the 6-dimensional CFT can be viewed as the insertion "Wilson surface"-operator [29 − 32]. In the probe approximation one can use a analogy between the Wilson loop in $N = 4$ SYM and the Wilson surface operators: The fundamental string is related to M2-brane probe. The D3 brane with electric flux and $AdS_2 \times S^2$ world-volume is related to a probe M5 brane with 3-form flux on its $AdS_3 \times S^3$ world-volume (with the S^3 embedded in the AdS_7). The D5 brane with electric flux and $AdS_2 \times S^4$ world-volume is related to a probe M5 brane with three form flux on its $AdS_3 \times S^3$ world-volume (with the S^3 embedded in the S^4). These probe branes and their supersymmetry where analyzed in [33, 34, 13]

The supergravity solutions we have obtained are the analog of the "bubbling" Wilson loop solutions [18, 12, 28]. They are fully backreacted and replace the probe branes by geometry and flux. In particular as the discussion of the $g = 1$ solution in section 3.4 showed there are two new nontrivial four cycles $C_{2,3}$ in the $g = 1$ solution. The fluxes through these cycles are the remnants of the probe M5-branes in the backreacted solution.

Unfortunately the $(2,0)$ theory for multiple M5-branes is not as well understood as $\mathcal{N} = 4$ SYM theory. It is possible that the bubbling solutions can be useful in the understanding of the M5-brane theory. It would be interesting to see whether there is an analog of the matrix model description of the BPS-Wilson loops (and its relation to the bubbling supergravity solution) for the Wilson surfaces.

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The general solution we have obtained has only one asymptotic $AdS_7 \times S^4$ region. It would be interesting to investigate whether its possible to have more than one asymptotic AdS region, this would presumably correspond to a harmonic function h with multiple poles. A similar phenomenon occurs in the case of half-BPS solutions which are asymptotic to $AdS_4 \times S^7$ which we are currently investigating [20].

Acknowledgments

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A. Proof of the regularity condition $W^2 > 0$

In this appendix we shall prove a theorem which is central to establishing the regularity of the general solution constructed in section 3.2.

Theorem 1. *When all η_n are equal to one another, the function G, defined by*

$$G = -\frac{1}{2}\sum_{n=1}^{g+1} \eta_n \left(\frac{1+i\alpha_n}{\sqrt{1+\alpha_n^2}} - \frac{1+i\beta_n}{\sqrt{1+\beta_n^2}} \right) \tag{A.1}$$

satisfies $W^2 > 0$, for all α_n, β_n subject to the ordering condition

$$\alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots < \alpha_g < \beta_g < \alpha_{g+1} < \beta_{g+1} \tag{A.2}$$

Numerical analysis suggests that this property holds, and also shows that, when not all η_n are equal to one another, the condition $W^2 \geq 0$ is violated for some range of α_n and β_n. We shall prove Theorem 1 for $\eta_n = +1$ for all $n = 1, \cdots, g+1$; the theorem for the opposite case $\eta_n = -1$ then follows immediately.

We begin by simplifying the condition $W^2 > 0$ as follows,

$$W^2 = -4|G|^4 - (G-\bar{G})^2 = \left(|G-\bar{G}| - 2|G|^2\right)\left(|G-\bar{G}| + 2|G|^2\right) \tag{A.3}$$

The second factor on the right hand side of the last equality is manifestly positive for all G, and may be dropped in the inequality. Thus, the condition $W^2 > 0$ becomes equivalent to the condition $|G-\bar{G}| - 2|G|^2 > 0$, which is equivalent to the following quadratic inequality

$$X^2 + \left(|Y| - \frac{1}{2}\right)^2 < \frac{1}{4} \qquad\qquad G = X + iY \tag{A.4}$$

where X, Y are real. In the sequel, it will be convenient to introduce the following notations,

$$\begin{aligned} p(\alpha) &\equiv -\frac{1}{2}\frac{1}{\sqrt{1+\alpha^2}} \\ q(\alpha) &\equiv +\frac{1}{2}\frac{\alpha}{\sqrt{1+\alpha^2}} \qquad\qquad p^2 + q^2 = \frac{1}{4} \end{aligned} \tag{A.5}$$

In terms of these functions, we define the following partial sums, for $m = 1, 2, \cdots, g+1$,

$$
\begin{aligned}
X_m &= \sum_{n=1}^{m} \Big(p(\alpha_n) - p(\beta_n) \Big) \\
Y_m &= \sum_{n=1}^{m} \Big(q(\beta_n) - q(\alpha_n) \Big)
\end{aligned}
\tag{A.6}
$$

so that the real and imaginary parts of G, defined in (A.4), are given by $X = X_{g+1}, Y = Y_{g+1}$.

The first key ingredient in the proof of Theorem 1 will be the fact that, for $\alpha \geq 0$, the functions $p(\alpha)$ and $q(\alpha)$ are strictly monotonically increasing as α increases.

A.1 The case $0 \leq \alpha_1$

We begin by proving Theorem 1 for the following special ordering,

$$
0 \leq \alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots < \alpha_g < \beta_g < \alpha_{g+1} < \beta_{g+1} \tag{A.7}
$$

Using the fact that $p(\alpha)$ and $q(\alpha)$ are monotonically increasing with α for $\alpha \geq 0$, it is immediate that $X_{g+1} < 0$ and $Y_{g+1} > 0$. Both bounds are sharp, as they can be saturated at the boundary of the domain (A.2) in the limit where $\alpha_n - \beta_n \to 0$. A lower bound for X_{g+1} and an upper bound for Y_{g+1} may be obtained by letting $\beta_n - \alpha_{n+1} \to 0$ (with $\alpha_{g+2} \equiv +\infty$). Putting all together, we obtain the following double-sided bounds,

$$
\begin{aligned}
0 < -X_{g+1} &< -p(\alpha_1) \\
0 < Y_{g+1} \quad &< \frac{1}{2} - q(\alpha_1)
\end{aligned}
\tag{A.8}
$$

To prove $W^2 > 0$, we proceed recursively. Using the definition (A.6), we have,

$$
\begin{aligned}
X_{g+1} &= X_g + p(\alpha) - p(\beta) \\
Y_{g+1} &= Y_g + q(\beta) - q(\alpha)
\end{aligned}
\tag{A.9}
$$

where we use the abbreviations $\alpha = \alpha_{g+1}$, and $\beta = \beta_{g+1}$. Notice that we have $X_g < 0$ and $Y_g > 0$. The quantity of interest is

$$
W_{g+1}^2 \equiv X_{g+1}^2 + \left(Y_{g+1} - \frac{1}{2} \right)^2 \tag{A.10}
$$

Here, we have suppressed the absolute value sign on Y_{g+1}, as we already know that $Y_{g+1} > 0$. To show that $W^2 > 0$ holds, it will suffice to show that $W_{g+1}^2 < 1/4$. Thus, we need to derive an optimal upper bound for W_{g+1}^2, and show that this bound is less than $1/4$.

We first derive an upper bound on W_{g+1}^2 as a function of α and β, subject to the condition that $\beta_g < \alpha < \beta$. To this end, express W_{g+1}^2 as follows,

$$
\begin{aligned}
W_{g+1}^2 &= (p(\beta) + x_g)^2 + (q(\beta) - y_g)^2 \\
x_g &= -X_g - p(\alpha) \\
y_g &= -Y_g + \frac{1}{2} + q(\alpha)
\end{aligned}
\tag{A.11}
$$

The bounds established earlier, namely $X_g < 0$ and $Y_g < 1/2$, guarantee that $x_g > 0$ and $y_g > 0$ for all values of $\alpha \geq 0$. We now search for the maximum of W^2_{g+1} as a function of β over the interval $\beta \in [\alpha, +\infty]$, with α viewed as fixed. To determine it, we investigate the derivative with respect to β,

$$(W^2_{g+1})'(\beta) = \frac{x_g \beta - y_g}{\sqrt{1+\beta^2}^3} \tag{A.12}$$

This derivative can vanish in the interval $\beta \in [\alpha, +\infty]$ if and only if $x_g\alpha - y_g \leq 0$. If this is the case, the corresponding point is $\beta_0 = y_g/x_g$, which should satisfy $\beta_0 > \alpha$.

Hence, the extrema of W^2_{g+1} as a function of β may be attained either at $\beta = \beta_0$, or at either one of the extremities of the interval $\beta \in [\alpha, +\infty]$. These three values are given by,

$$\begin{aligned} W^2_{g+1}(\beta_0) &= x_g^2 + y_g^2 + \frac{1}{4} - \sqrt{x_g^2 + y_g^2} \\ W^2_{g+1}(\alpha) &= x_g^2 + y_g^2 + \frac{1}{4} + 2p(\alpha)x_g - 2q(\alpha)y_g \\ W^2_{g+1}(\infty) &= x_g^2 + y_g^2 + \frac{1}{4} - y_g \end{aligned} \tag{A.13}$$

Since $x_g, y_g > 0$, it is manifest that $W^2_{g+1}(\beta_0) < W^2_{g+1}(\infty)$. Thus, $W^2_{g+1}(\beta_0)$ cannot be the optimal upper bound for $W^2_{g+1}(\beta)$. Comparing the remaining two possible values, we find,

$$W^2_{g+1}(\infty) - W^2_{g+1}(\alpha) = 2p(\alpha)X_g + 2\left(1 - q(\alpha)\right)Y_g \tag{A.14}$$

Given that $X_g < 0$ and $Y_g > 0$, it follows that the right hand side is positive and so that $W^2_{g+1}(\infty)$ is the optimal upper bound. In summary,

$$\begin{aligned} W^2_{g+1} &< V_g(\alpha_{g+1}) \\ V_g(\alpha_{g+1}) &\equiv \Big(X_g + p(\alpha_{g+1})\Big)^2 + \Big(Y_g - q(\alpha_{g+1})\Big)^2 \end{aligned} \tag{A.15}$$

for all values of α_{g+1} such that $\beta_g < \alpha_{g+1}$.

Since $X_g < 0$ and $Y_g > 0$, it is straightforward to derive an upper bound for the right hand side of (A.15). Indeed, both terms increase as α_{g+1} decreases. Thus, the optimal bound for the right hand side is attained when α_{g+1} assumes its smallest possible value, which is $\alpha_{g+1} = \beta_g$. Hence, we have

$$V_g(\alpha_{g+1}) < V_g(\beta_g) \qquad\qquad \alpha_{g+1} \in [\beta_g, \infty] \tag{A.16}$$

But, using the definitions of X_g and Y_g in terms of α_n and β_n, we see that the quantity $V_g(\beta_g)$ admits a drastic simplification,

$$V_g(\beta_g) = \Big(X_{g-1} + p(\alpha_g)\Big)^2 + \Big(Y_{g-1} - q(\alpha_g)\Big)^2 = V_{g-1}(\alpha_g) \tag{A.17}$$

Combining all, we get a recursive series of bounds,

$$W^2_{g+1} < V_g(\alpha_{g+1}) < V_{g-1}(\alpha_g) < V_{g-2}(\alpha_{g-1}) < \cdots < V_0(\alpha_1) \tag{A.18}$$

From their definitions, $X_0 = Y_0 = 0$, we readily find $V_0(\alpha_1) = 1/4$, so that $W^2_{g+1} < 1/4$. This concludes the demonstration of Theorem 1 for the case $0 \leq \alpha_1$.

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A.2 The case $\beta_{g+1} \le 0$

Next, we proceed to proving Theorem 1 for the following special ordering,

$$\alpha_1 < \beta_1 < \alpha_2 < \beta_2 < \cdots < \alpha_g < \beta_g < \alpha_{g+1} < \beta_{g+1} \le 0 \tag{A.19}$$

It is not necessary to repeat the steps analogous to the proof for the case $0 \le \alpha_1$, since we can reduce the present case to the $\alpha_1 \ge 0$ case by changing variables,

$$\begin{aligned} \alpha_n &= -\tilde{\beta}_{g+2-n} \qquad\qquad n = 1, \cdots, g+1 \\ \beta_n &= -\tilde{\alpha}_{g+2-n} \end{aligned} \tag{A.20}$$

The $\tilde{\alpha}_n$ and $\tilde{\beta}_n$ are now all positive and satisfy the following ordering,

$$0 \le \tilde{\alpha}_1 < \tilde{\beta}_1 < \tilde{\alpha}_2 < \tilde{\beta}_2 < \cdots < \tilde{\alpha}_g < \tilde{\beta}_g < \tilde{\alpha}_{g+1} < \tilde{\beta}_{g+1} \tag{A.21}$$

Denoting the corresponding function by G^-, and its real and imaginary parts by X^-_{g+1} and Y^-_{g+1}, we have by definition,

$$\begin{aligned} X^-_{g+1} &= \sum_{n=1}^{g+1} \Big(p(\tilde{\beta}_n) - p(\tilde{\alpha}_n) \Big) = -\tilde{X}_{g+1} \\ Y^-_{g+1} &= \sum_{n=1}^{g+1} \Big(q(\tilde{\beta}_n) - q(\tilde{\alpha}_n) \Big) = +\tilde{Y}_{g+1} \end{aligned} \tag{A.22}$$

where $\tilde{X}_{g+1}$ and $\tilde{Y}_{g+1}$ are given by (A.6) but with $\alpha_n \to \tilde{\alpha}_n$ and $\beta_n \to \tilde{\beta}_n$. From the proof of the case $\alpha_1 \ge 0$, it now follows that $W^2 > 0$ also for this special case.

A.3 The general case

Next, we shall prove Theorem 1 for the cases whose ordering is given by

$$\alpha_1 < \beta_1 < \cdots < \alpha_N < \beta_N \le 0 \le \alpha_{N+1} < \beta_{N+1} < \cdots < \alpha_{g+1} < \beta_{g+1} \tag{A.23}$$

for $N = 1, \cdots, g$. (The proof for the case with the ordering $\cdots < \alpha_N < 0 < \beta_N < \cdots$ follows the same steps, or may be derived by taking the limit $\alpha_{N+1}, \beta_N \to 0$ in the ordering of (A.23), and need not be detailed here.)

It remains only to prove Theorem 1 for the ordering (A.23). To do so, we use the fact that the behaviors of the variables larger than 0 and those smaller than zero are independent of one another. Concretely, we define the following partial sums,

$$\begin{aligned} X^- &= \sum_{n=1}^{N} (p(\alpha_n) - p(\beta_n)) \\ X^+ &= \sum_{n=N+1}^{g+1} (p(\alpha_n) - p(\beta_n)) \\ Y^- &= \sum_{n=1}^{N} (q(\beta_n) - q(\alpha_n)) \\ Y^+ &= \sum_{n=N+1}^{g+1} (q(\beta_n) - q(\alpha_n)) \end{aligned} \tag{A.24}$$

so that the full sums are given by

$$G = X_{g+1} + iY_{g+1} \qquad\qquad \begin{aligned} X_{g+1} &= X^+ + X^- \\ Y_{g+1} &= Y^+ + Y^- \end{aligned} \tag{A.25}$$

To the sums X^+, Y^+, we apply the results derived for case $0 \leq \alpha_1$, while to the sums X^-, Y^-, we apply the results derived for case $\beta_{g+1} < 0$, namely

$$\begin{aligned} X^+ &< 0 \qquad 0 < Y^+ < \frac{1}{2} \qquad\qquad (X^+)^2 + \left(Y^+ - \frac{1}{2}\right)^2 < \frac{1}{4} \\ X^- &> 0 \qquad 0 < Y^- < \frac{1}{2} \qquad\qquad (X^-)^2 + \left(Y^- - \frac{1}{2}\right)^2 < \frac{1}{4} \end{aligned} \tag{A.26}$$

From the fact that X^+ and X^- have opposite sign, and the fact that $Y^+ - 1/2$ and Y^- have opposite sign, it follows immediately that

$$\big(X^+ + X^-\big)^2 + \left(Y^+ + Y^- - \frac{1}{2}\right)^2 < \big(X^\pm\big)^2 + \left(Y^\pm - \frac{1}{2}\right)^2 < \frac{1}{4} \tag{A.27}$$

so that $|G - \bar{G}| - 2|G|^2 > 0$, and thus $W^2 > 0$, which completes the proof of Theorem 1 in the general case. Note that the range of G in the general case is all of the disc $|G - 1/2| < 1/2$.

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